

Simple Algorithm for rigorous least squares solution to levelling networks

SAMI H. AL I

DEPT. of Civil Engineering, College of Engineering, University of Tikrit, Tikrit, Iraq

Abstract

Simple Algorithm for rigorous least squares solution to Levelling networks

The theoretical and practical aspects of rigorous least squares solution to levelling networks are investigated by a simple algorithm which developed in this work. Experiment indicates that levelling networks data which involve field survey differential leveling observations i.e., difference in heights between the existing benchmark and new established junction points, can be rigorously adjusted without significant increase to the costs as compared to non-rigorous or empirical adjustment methods.

The algorithm is based on the method of variation of coordinates (observation method).

The final output allows a posteriori variance-covariance analysis. It is shown that because of its simplicity, this algorithm provides a universal use capability in ground plane surveying for measuring and computing the adjusted heights of the new established benchmarks which are used for roads, sewage, irrigation, geology and other civil and environmental engineering projects.

Introduction

The technological evolution in engineering surveying, in terms of measuring instruments and computing devices, has provided the surveyor with more redundant measurements and almost unlimited computation capabilities at a relatively low cost. There are many reports and references, however, related to the mathematical formulation and algorithm analysis of vertical control network problems. A few examples of such reports are: Bomford (1), Bomford (2), Ashkenazi (3), Ashkenazi (4), Uotila (5), Barber (6), Clark (7), Hazay (8), Rainsford (9), ALLAN et al (10), Richardus (11), Milne (12), Moffitt (13), Mikhail (14), Mikhail et al (15), Mikhail et al (16), Wolf (17), and Schofield (18). A look at these reports reveals that a significant amount of work, mainly oriented to the large geodetic vertical control networks for establishing bench marks, is being done.

Vertical control network problems, considered in this paper, are problems which deal with the determination of the height coordinate of a number of new bench marks in a surveying levelling network (i.e. plane surveying applications) by using direct observations of differential levelling which are measured by spirit, automatic (self compensating), precise and digital levels. If difficulties arise in connecting networks across rivers, a reciprocal simultaneous levelling is used (Bomford (1), Kavanagh et al (19) and Nathanson et al (20)). Plane surveying applications include the establishment of the new bench marks (junction points) for the second, third and fourth order vertical (levelling) control network, with special applications to roads, sewage, irrigation, geology and other civil and environmental engineering projects.

There are various methods for adjusting the vertical control network, i.e. the approximate (non-rigorous) and rigorous least squares methods. Despite the simplicity of the approximate methods, but they do not provide information on the accuracy of the final coordinates and can thus be considered as a most serious drawback. But, the rigorous least squares

adjustment allows a better statistical analysis of the accuracy of the survey network.

Therefore, the rigorous least squares solution is superior to any other adjustment method in surveying in determining simultaneously, with maximum likelihood,

all the unknown parameters of the problem, and providing scientific control over the precision.

The precision can be significantly improved by the redundancy and the geometric strength of the figure. Each observation contributes to the final solution according to its precision. The precision of an observation depends on the type of measuring equipment and the skill of the surveying team.

Another advantage of a rigorous solution is that it is possible to detect blunders among the observations or among the control (existing bench marks) points by applying appropriate statistical tests (Uotila (5)).

The rigorous least squares adjustment may be done by either a parametric method (observation equations method) or a conditional method (Bomford (1), Hazay (8), Rainsford (9), Richardus (11), Mikhail (14), Mikhail et al (15), Mikhail (16), Wolf (17), Thompson (21), and Gale (22)).

In general, the first and second order vertical control networks are usually adjusted by the least-squares method, although differences in heights adjusted approximately or rigorously are within the accuracy of the observations. The least-squares adjustment of the levelling networks is much simpler than the adjustment of horizontal networks. The linear observation equations or linear condition equations can be formed from the observed quantities .

In the current research, only the observation equations (variation of coordinates) (or differential displacements) method was used (Schofield (18), Milne (12), Atkinson et al (21), Mikhail et al (15) and Mikhail et al (16)) .

This paper, however, will discuss a simple algorithm to obtain a rigorous least squares for vertical control networks, by using the adjustment computer program ADJLEV. This simplicity of this algorithm consists of: (a) simple mathematical formulation based on the variation of coordinates, (b) simple determination of the geometric configuration of the leveling network with an easy coding system. The input data for the computer program is mainly composed of the observations, minimizing the use of control instructions. The solution of the problem is carried out by iterations with approximate values originally assigned to the parameters

(computed from the original levelling measurements) with a priori estimated variance.

The adjustment program ADJLEV was written in standard FORTRAN IV and run on compiler 90.

The program consists of a series of a relatively small , well -defined modules (subroutines and functions) to allow easy handling , testing and manipulation. Each module was written to perform a specific task. A small program was designed as a connecting program in order to connect together the necessary subroutines.

The output of these programs is either in the form of computer printout, softcopy on a visual graphic display terminal (e.g. CRT), graphic output on an automatic plotter or store on a disc file for subsequent plotting and manipulation.

A generalised flow chart indicates the main stages of the program adjustment operations which is shown in Figure (1).

Control Survey

In most types of survey, however, large or small , the normal survey principle is to work the " whole " to the " part ". Therefore, a basic control network has to be established in and around the survey area.

The purpose of the control network is to provide a uniform framework of reference for the coordination of all survey operations of a survey area. Subsequent surveys will be fitted inside this network and adjusted to it.

The accuracy of the control network should be higher than the subsequent surveys. However, the accuracy of the control network should satisfy the purpose of the survey. With regard to the cost of a survey, different configuration with different methods of observation and equipment can achieve the required accuracy, but the cheapest design will be selected.

The density of a control network depends upon the purpose of the network and to some extent upon the availability of the instruments to carry out the survey. The control network may be tied to an existing control network or independent of it.

Control survey consists of horizontal and vertical control networks. However, different methods of surveying are required to establish each control network. Although, some of the control points may be common to both control networks.

It is common practice in surveying to treat the planimetric model (plan) separately from the levelling, because the planimetric positions are not related to the shape of the ground surface. Also the accuracy requirements of heights may be higher than for planimetric details. Although, in a survey area there are common points for plan and height.

In the current research, only a brief description of the vertical control network will be considered.

The vertical control network is used to provide control for the spot heights and contouring of a specific survey area.

The vertical control network can be surveyed by different ground surveying methods such as, the well-known method of differential levelling using spirit, automatic (self-compensating) or digital levels, trigonometrical heighting or altimetry (Bomford (1) , Kavanagh (19), Allan et al (10) and Clark et al (7)).

If difficulties arise in connecting networks across rivers, a reciprocal simultaneous levelling is used. In this project , differential leveling method was used.

An excellent review of levelling instruments and instrumental errors is given by DEMLICH (24).

Any general manual on surveying gives adequate information on field procedures and the basic adjustment of the instruments .

A review and summary of results of investigations on sources of errors and measuring techniques with different models of automatic levels and digital levels is given by (NATHANSON et al(20) , KOWALCZYK (25) and COOPER (26)). Levels may be based on an existing bench marks of known heights in the survey area or on a local temporarily bench marks of assumed heights.

The network should be of uniform quality, and the heights should be referred to the same reference level.

Generally, the accuracy of levelling is influenced by random and systematic errors of measurements (Bomford (1)). The standard error of levelling σ may be expressed as a function of the length S of the levelling line in the form

$$\sigma = \sqrt{e^2 S + \delta^2 S^2}, \quad (1)$$

where e is the random component per unit distance and δ is the systematic component unit distance.

The accuracy of leveling σ may be characterized by a standard deviation σ_0 per unit distance with a propagation over the distance S expressed by

$$\sigma = \sigma_0 \sqrt{S} \quad (2)$$

As a general rule, levelling of each section of the network is done twice; therefore, σ_0 characterizes the accuracy of double levelling.

Specifications for field procedures are usually based on a maximum permissible discrepancy between two independent levellings of the same line. If the maximum error of a double leveling is accepted as equal to $2 \sigma_0 \sqrt{S}$ (at 95% confidence level) (Ref. 28), the maximum discrepancy Δ between two levellings is equal to

$$\Delta = 4 \sigma_0 \sqrt{S} \quad (3)$$

Where : σ_0 = standard deviation per unit distance

S = length of the levelling line in kilometers.

If the discrepancy is larger than is permitted, the levelling of the line must be repeated.

Moreover, the following values of Δ from national specifications of Canada (Ref. 28) are as follows :

$$\text{first order } \Delta = 4 \sqrt{S} \text{ mm}$$

$$\text{second order } \Delta = 8 \sqrt{S} \text{ mm}$$

$$\text{third order } \Delta = 24 \sqrt{S} \text{ mm}$$

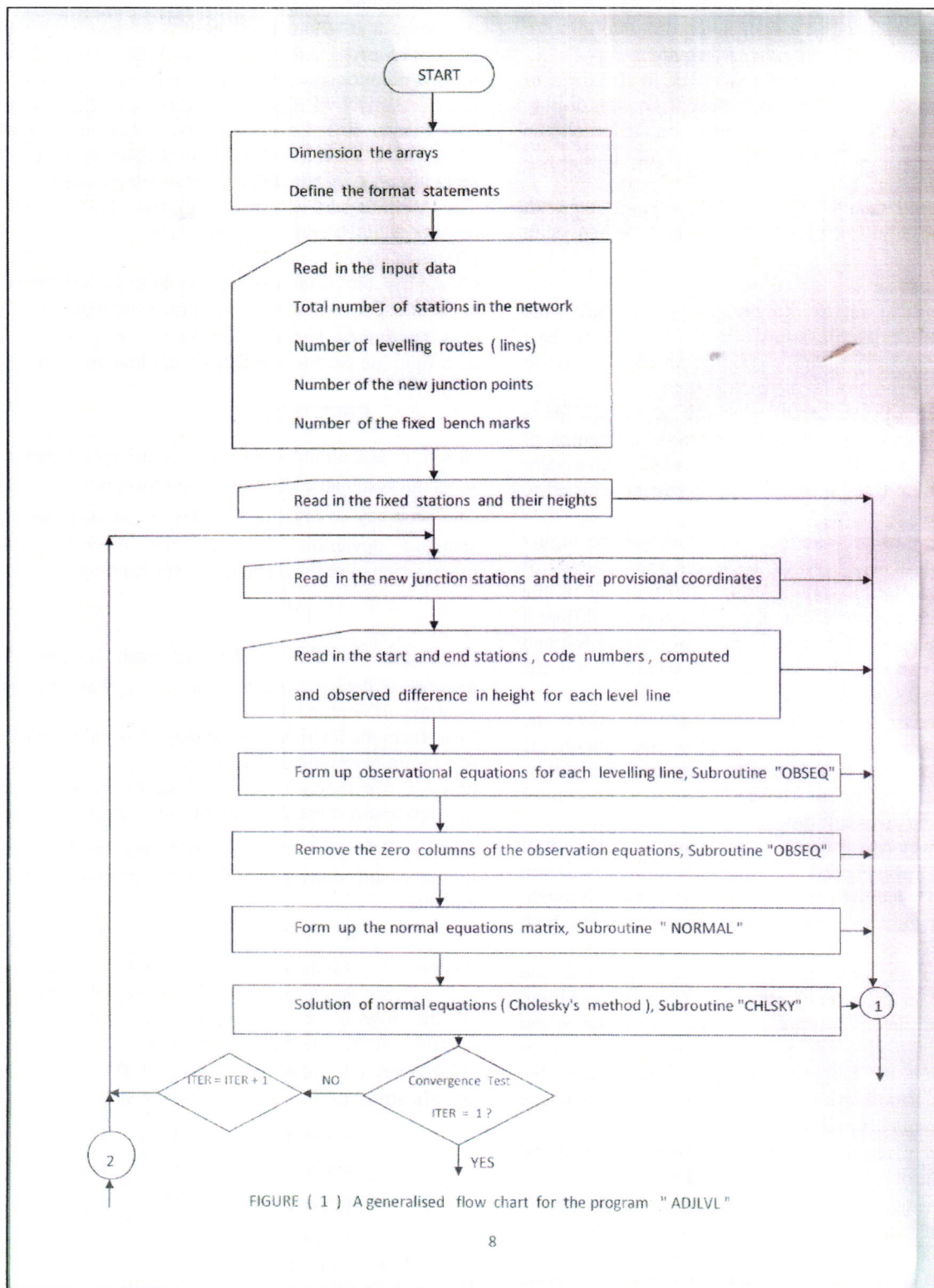
$$\text{fourth order } \Delta = 120 \sqrt{S} \text{ mm},$$

where S is in kilometers.

However, most civil and environmental engineering projects (vertical control networks) are classified as a second and third order accuracy, while geology projects nets are classified as third and fourth order accuracy.

An example (taken from Ref. (27)) of a network with four junction points is given in Figure (2). Mean differences in heights of 14 lines are shown in metres, distances (in parentheses) in kilometers, and heights of four fixed (bench marks) points in metres. Level lines for this network were independently measured in both directions (double levelling).

This example is used as a source of data in order to test the deigned current program 'ADJLVL'. Other test data was also used from other sources , but because of the lack of the space of the current paper was not included. However, the preparation of the input data for the program 'ADJLVL' will be considered in the next section (See Section 3) .



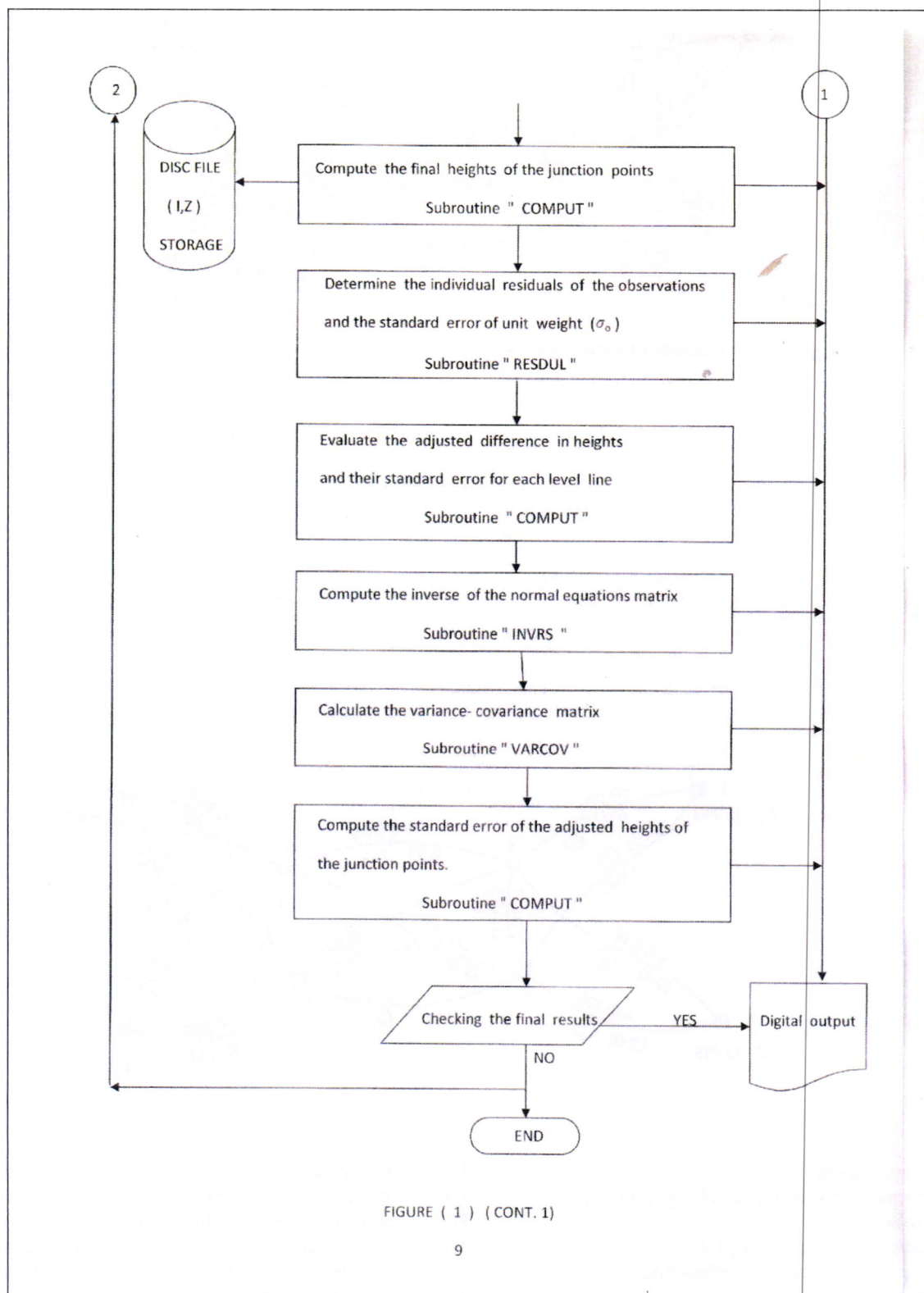


FIGURE (1) (CONT. 1)

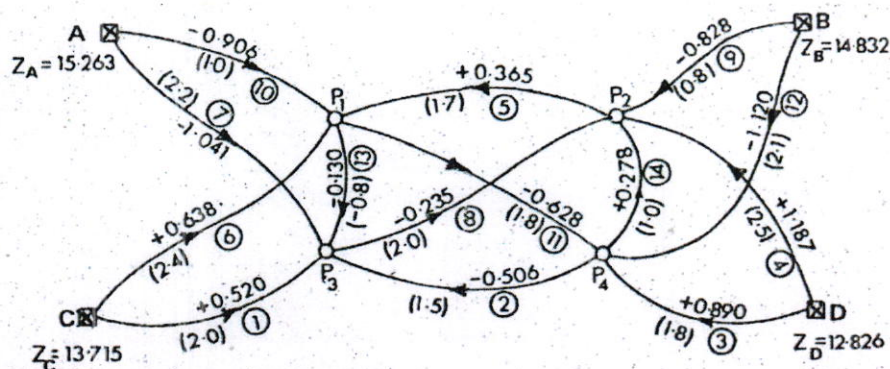
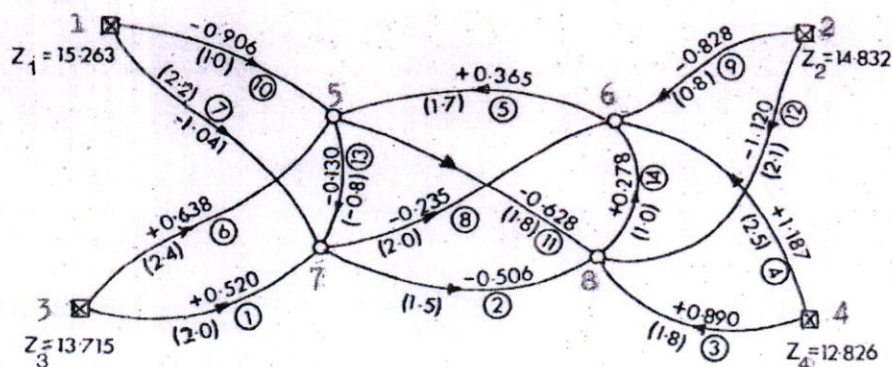


Figure (2) Example of a leveling network.



3. Data preparation

The data preparation for the levelling network can be divided as follows :

- (a) preparation of a large scale plot.
- (b) Selection of the route of levelling lines.
- (c) Coding system
- (d) Punching and verifying the field survey data via a terminal.

(a) preparation of a large scale plot

It is normal practice to prepare a large scale plot (e.g. scale of 1:2000 or larger) of the measured vertical network model for purposes of coding the levelling points, reduced field observations, coordinates of fixed points and numbers of stations, levelling lines and the direction of the positive height ; preparation of the input data for the processing and checking procedure (See Figure (2) and Figure (3)).

The next step is to choose the coordinate reference system for the leveling network.

The vertical position (height) requires only a one-dimensional coordinates i.e. the height of a point above a reference datum (e.g. the mean sea level or assumed datum). Since the heights are relative to one another, any point is chosen as a datum and the other values are referred to it.

(b) Selection of the route of levelling lines

The process of selecting the route of each levelling line can be performed before the execution of the field observations or later in the office. In the later case, the selection of a particular should be made for subsequent processing, storing and plotting.

Each level line (route) must be start with a fixed bench mark (known height) and end with a junction point

(approximate height) or the level line starts with a junction point and finishes with other junction point.

(c) Coding system

Generally, when using computer techniques, it is common practice to adopt some form of coding notation to represent information. This is done in order to avoid ambiguity, reduce data preparation to minimum, reduce computer storage costs and computer processing.

Therefore, all survey data recorded in the field for subsequent use by computer must be correctly referenced for purposes of identification, processing and plotting.

The surveyor should choose the appropriate coding system. However, the most suitable form of field coding depends on the degree of the automation that is required for subsequent processing and plotting.

The coding can be carried out in the field or subsequently in the office aided by the surveyor and his field notes and sketches. In the former method, a considerable saving in time can be achieved by avoidance of transcription time and the generation of transcription errors. However, some surveyors prefer to use the later coding (transcoding) method to suite their own existing software methods.

Two coding systems can be used in practice, the first is to employ only numeric codes and the other (alphanumeric or alphabetic) character codes.

The choice of a specific coding system depends upon the computer circumstances (i.e. the computer language and the amount of storage space available in the computer to be used) and upon personal preference.

The advantage of a numeric coding system is more efficient in computer terms because it requires less data, less storage and faster processing; these factors will reduce the cost. But, the disadvantage of such system is difficult to remember from the point of view of the user.

The (alphanumeric or alphabetic) characters coding system is quick for the layman to understand, user friendly, and has a wide variety of coding possibilities. However, it can be translated into numbers by a suitable computer program, but this conversion process may possibly introduce errors (Day (29, 30) in the current research, the numeric coding is used for coding the levelling network).

The criteria for designing such numbering system are, it should be simple, unique, flexible and of uniform format (David (31), Bogaerts (32), Blachlut et al (33), Palmer (34), Johnston (35), Mils et al (36), Sherwood (37) and Kavanagh (19)).

In the present system, the numbering system is kept as simple as possible and the number of digits for each code number is kept as small as possible in order to allow easy handling, checking and avoiding errors; where checking and correcting a large amount of data are relatively costly operations.

The way followed in this paper is to define by appropriate encoding method. The location of networks stations and network lines which compose the surveying levelling network.

The coding method is illustrated in the following example. In Figure (3), a surveying levelling network is presented with 4 junction stations, 4 bench marks, and 12 levelling lines.

The network is first prepared by employing the following numbering process system:

1- Numbering the bench marks first in any order but using sequential numbers starting with one (See Figure (3)). Single alpha symbols are also assigned to the fixed benchmarks only but the alpha symbols are not used in the preparation of the input data of the current program. It is used in the preliminary computations for coding the bench marks and junction points only (See Figure (2)).

2- Numbering the junction stations in any order but starting with the last sequential number plus one of the last benchmarks in the network.

For example, the total number of the fixed benchmarks are four as shown in Figure (3), then the first sequential number of the junction points should be assigned five (i.e. the sequential numbers of the junction points will be as follows 5,6,7,8 ... etc.).

3- Numbering and directing all the levelling lines. The leveling lines (routes) were numbered in any order using sequential numbers starting with one.

Also arrows indicate the directions of the leveling lines i.e. the direction of the positive (increasing) height. Distances (in parentheses) in Kilometers of each route line are shown in the Figure (3). This is a useful rule to be adopted as it helps in getting signs right. Also, no observed height difference is omitted nor is one listed twice. In this way,

by using a simple numbering system, the geometric configuration of the network is well established.

(d) Punching and verifying the field survey data via a terminal

Differential leveling survey data were abstracted from the field book with other necessary data are punched via a personal computer (P.C). They are used as input data for the adjustment computer program.

The data is keyed via a computer keyboard and converted directly onto disc file. At this stage, the data can be checked and edited by using the computer's editing facilities. This system provides a very rapid method of detecting errors in the input data, and allows their speedy corrections, and no material is wasted, where the input data can be visually checked on the computer's screen or the can be printed out for visual checking against the original field data.

The input data are read in and tested for errors and inconsistencies. Also, error messages are incorporated into the program in order to check the input data parameters against their limits (test values).

In the current computer program, free format input data facility was used which is very convenient for interactive program. In free format, input data is represented with items separated by commas and/or space. Free format output can also be used.

The problem of handling the large amount of field measurements, and the data management in general, is becoming more serious than the adjustment procedure itself. Much time may be wasted if the data are not properly checked and fed into the computer.

Nowadays, with the advent use of the electronic field book (e.g. Geodat) the data can be transferred directly into the computer for subsequent processing and plotting. This will reduce coding, copying, keyboard and transcription errors. The data collector can be used with

the digital level (e.g Leica digital level) (Kavanagh et al (19) and Nathanson et al (20)).

4. Preliminary computations and choice of adjustment method

Adjustment of levelling networks may be divided into four groups :

1. In order to determine the height of a point , lines of levels are led to the new junction point from several points of fixed heights (existing bench marks) (See Figure (4)).
 2. Starting from a point of fixed height (existing bench marks), a line of levels is conducted through one or more junction points with heights to be determined , to another point of fixed heights (See Figure (5-a)), or returning to the initial point itself, a closed (loop) circuits of levels is established (See Figure (5 - b)).
 3. A whole system of lines of levels is developed between more points of fixed heights (existing bench marks) and more new junction points (See Figure (6)).
 4. Circuits of levels are joined with one another. Among the points include in the circuits more junction points included in the circuits more point of fixed heights (existing bench marks) may occur (See Figure (7)).
- However, the ends of a line of levels are called " junction points". All the above groups of the levelling network

will be included in the current computer program. The method of calculations is to obtain the adjusted values (heights) for these junction points.

During the vertical control network adjustment, the network points counted are only those bench marks that already have fixed values of heights and those new points (junction points) from which at least one or more levelling lines lead to other network levelling points .

If there are no fixed points (Bench marks), one network point must be selected as the initial point with an arbitrarily fixed height and the other values are referred to it because, heights are relative to one another.

A levelling network consists of a number of levelling lines (traverses) that are run between the junction bench marks and fixed points of the higher order accuracy network.

In preparatory computations for the network adjustment are as follows :

1. The field survey data for levelling was abstracted from the traditional field differential levelling (See Section 3 and Figure (2)).
2. Each levelling line is treated separately as a unit of computation. The preliminary check calculations for the leveling network are tabulated in Table (1).

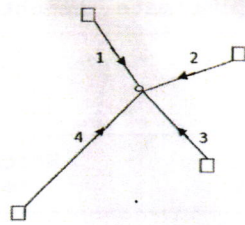


FIGURE (4)

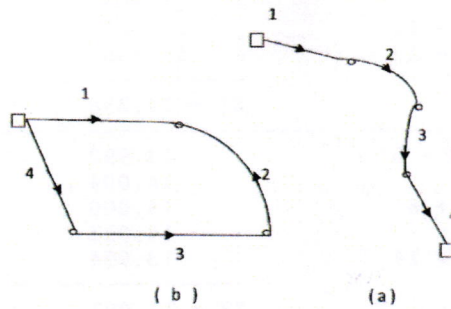


FIGURE (5)

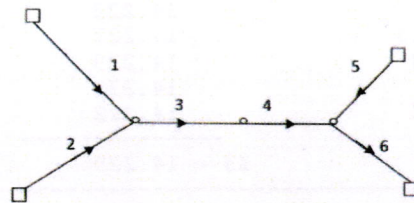


FIGURE (6)

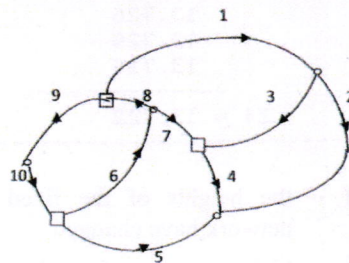


FIGURE (7)

- Existing Bench Marks
- New Junction Points

TABLE (1) Results of the approximate heights of the junction points

Junction point	Route of calculations	Height (m)	Deviation (mm)
1	A + 10	14.357	+1
	B + 9 + 5	14.369	-11
	C + 6	14.353	+5
	D + 3 - 11	14.344	+14
	C + 1 - 13	14.365	-7
Z1 = 14.358			
2	A + 10 - 5	13.992	+9
	B + 9	14.004	-3
	C + 1 + 8	14.000	+1
	D + 4	14.013	-12
	D + 3 + 14	13.994	+7
Z2 = 14.001			
3	A + 7	14.222	+7
	A + 10 + 13	14.227	+2
	B + 9 - 8	14.239	-10
	C - 1	14.235	-6
	D + 3 - 2	14.222	+7
Z3 = 14.229			
4	A + 10 + 11	13.729	-7
	B + 12	13.712	+10
	B + 9 - 14	13.726	-4
	C + 1 + 2	13.729	-7
	D + 3	13.716	+6
Z4 = 13.722			

3. Mean differences h_i of heights from repeated and accepted levellings are calculated for each line.

The means are tabulated and written on a large scale sketch of the network. Moreover, arrows are used to indicate the directions of the computations. The sketch should also show numbering of the leveling lines, length S_i of the lines and heights Z_i of the higher order bench marks (See Section (3)).

An example (taken from Ref. 27) of a vertical network with four junction points is given in Figure (2). Mean differences in heights of 14 levelling lines are shown in metres, distances (in parentheses) in kilometers, and heights of the four fixed points in metres.

The adjustment program was tested with various differential levelling observations which are used to establish bench marks for sewage, irrigations and geology schemes, but, because of the lack of space of the current paper are not included.

4. Approximate heights of the junction points are computed on several different routes from the fixed points (See Figure (2) and Table (1)). As a rule, one should use as many routes of calculations as levelling lines joining the junction points.

The spread of the results and a comparison of the means with individual approximate heights serve a further check on the quality of the measurements and indicate whether

the heights of the fixed points of the higher order network have changed.

If the deviations from the mean are within $2\sigma \cdot \sqrt{5}$ (See Eqn. (2)), the field results and the heights of the fixed points may be accepted for the final computation of the coordinates of the junction points.

In the example, the desired accuracy of leveling was $\sigma \leq 4\text{mm/km}$. All deviations in Table 1 are within the value of $8\text{mm} \cdot \sqrt{5}$ (i.e., second order network according to the standard specifications) (Ref. (28)) (See Section 3).

The final calculation of heights of the junction points may be done by either the least squares or another approximate adjustment.

The approximate method of adjustment follows the procedure shown in Table 1 where, the adjusted heights of junction points are calculated not as the arithmetic means but as weighted means with weights equal to $W_i = 1/S_i$, where S_i is the length of the calculation route. Heights of other bench marks along each individual level line are then calculated by an interpolation of the misclosure between the adjusted coordinates of the junction points and the field results of the differences in height.

However, the approximate method of adjustment do not provide information on the final adjusted coordinates and

can thus be considered as a most serious drawback. But, the least squares adjustment allows a better statistical evaluation of the accuracy of the network (See Section (1)).

The least squares adjustment of the leveling networks is much simpler than the adjustment of the horizontal networks. The linear observation equations (in the parametric method) or linear condition equations (in the conditional method) can be formed directly from the observed quantities .

The rigorous least squares adjustment may be done by either a parametric method (observation equations method) or a conditional method (correlate method).

Nowadays, the method of condition equations is not often used because the method of observation equations is usually easier to programme. Moreover, the formulation of conditions of adjustment, may be difficult , if not impossible, to formulate in a complex network containing many redundancies (Schofield (18)).

In the current research, only the observation equations method was used and will be reviewed in the next section (See Section (5)). In survey adjustments, this method is sometimes referred to as variation of coordinates (or differential displacements) method (Bomford (1), Hazay (8), Richardus (11), Milne (12) and Mikhail et al (15)).

5. Least-Squares Adjustment (Parametric)(variation of coordinates) Method

Each levelling line of a levelling network between point I and j produces one linear observation equation of the form :

$$d z_i + d z_j + h_c - h_o = V \quad (4)$$

Where $d z_i$ and $d z_j$ are unknown corrections (parameters) to the approximate coordinates (heights) of the new points,

h_c = is a difference in height between points i and j calculated from the approximate heights of the new points and given heights of the fixed points, h_o = is the observed difference in height, V = a residual which is a correction to h_o after adjustment.

If the leveling line begins or ends on a fixed points, the value of $d z_i$ or $d z_j$ equals to zero.

In general , the variation of coordinates method produces a set of observation equations.

A set of observation equations may be written in the matrix form as :

$$AZ + L = V \quad (5)$$

Where A is the matrix of the coefficients of the unknowns parameters in the observation equations, Z is a column matrix of the unknown parameters $d z_i$ and $d z_j$, L is a column matrix of the difference ($h_c - h_o$) i.e. ' computed minus observed ' differences in height, and V is a column matrix of the residuals (corrections).

The principle of least squares adjustment may be stated briefly as follows (Thompson (22),Gale (23) and Moffitt (13)) :

1. For equal – weight measurements :

The sum of the squares of the observational residuals (corrections) must be a minimum.

$$\text{i.e. } \sum V^2 = \text{minimum} \quad (6)$$

or in matrix forms, the quadratic form :

$$V^T V = \text{minimum}$$

Assuming , that all measurements are uncorrelated and equal precision (weight) .

2. For unequal-weight measurements

The sum of the squares of the weighted residuals (corrections) is a minimum, i.e.

$$\sum W V^2 = \text{minimum} \quad (7)$$

or in matrix notations, the quadratic form :

$$V^T W V = \text{minimum}$$

In which V is a column matrix of n residuals, and V^T is the transpose of V , and W is A square diagonal ' weight ' matrix of the inverse of the variances of the observations

$$W = \begin{bmatrix} w_1 & 0 & \dots & 0 \\ 0 & w_2 & & \vdots \\ \vdots & \vdots & & 0 \\ 0 & 0 & \dots & w_n \end{bmatrix} \quad (8)$$

Where n is the number of observations or measurements. This weight matrix of the observations (W) assumes that measurements to be adjusted are not correlated (correlation free) and different weight .

For vertical (levelling) control network, empirically, the weight is inversely proportional to the to the respective levelled distances or the number of setups of the instrument (Kavanagh (19)); where there is a good correlation between the number of setups and the surveyed distance.

Therefore, the weights of observations for each level line are usually computed as :

$$W_i = 1/S_i \quad (9)$$

$$\text{Or } W_i = 1/NS_i \quad (10)$$

Where S_i = the levelled distance between the two height points, and NS_i = the number of setups of the instrument between the two height points.

Each observation equation is multiplied by the square root of the corresponding weight :

$$\sqrt{W} A Z = \sqrt{W} L \quad (11)$$

So that the multiplication by the square root of the weight reduces all equations to zero dimension (dimensionless).

The element of the matrix L i.e. (computed - observed) ($C - O$) of the differences in height, where the computed (provisional) heights are determined from the preliminary computations (See Table (1))

Employing the least squares principle for the of set observation equations (Eqn. (5)). In other words to minimize the quantity $V^T V$ or $V^T W V$ (See Eqn.(6) and Eqn. (7)). The final result of this process is the normal equations which are as follows (Thompson (22)and (Gale (23):

$$A^T W A Z = A^T W L \quad (12)$$

$$\text{or } N Z = K \quad (13)$$

$$\text{Where } N = A^T W A$$

$$K = A^T W L$$

However, prior to the formation of normal equations matrix, the zero columns corresponding to the unknown corrections (Z) at each fixed station (Existing Benchmarks) are removed from the observation equations matrix; if the columns of zero terms are not removed, the resulting normal equations matrix (N) would be singular.

The normal equations matrix (N) is a symmetrical and positive definite. The well known Cholesky's method was used to solve the normal equations matrix (N).

Cholesky's method is the most suitable method for programming, storage and precision of the results (Rainsford (9), Thompson (39), Bomford (1), Ashkenazi (3) and Rice (38)).

The unknown parameters Z are calculated from :

$$Z = -N^{-1}K \quad (14)$$

Thereafter, The adjusted coordinates (heights) of the new junction benchmarks are calculated.

Where, the approximate heights of the new points are accepted from the preliminary calculations (See Table (1)).

The corrections (residuals) V_i to the measured height differences in heights can now be obtained by substituting the value of Z into the observation equations (Eqn. (2)) (with zero columns removed) to get a value for ($\sqrt{W} V$) for each observation, then each residual (V) can be obtained by dividing ($\sqrt{W} V$) by $\sqrt{W}$.

The values of the matrix V are used in the computations of an estimated standard error of unit weight of the level network

$$\hat{\sigma}_0 = \sqrt{\frac{\sum W V^2}{n-u}} \quad (15)$$

In which n is the number the observed leveling lines and u is the number of junction points.

The quantity (n-u) is the number of redundant measurements or the number of degrees of freedom (DF). Since , the weights were computed from Eqn. (9) with S in kilometers.

The standard error of unit weight ($\hat{\sigma}_0$) is used as indication of the quality of the adjustment.

The computed value of $\hat{\sigma}_0$ represents an estimated standard deviation of leveling per Kilometer.

An independent check is performed by computing the following equation :

$$A^T W V = 0 \quad (16)$$

This arithmetical check is used as a check for the formation of normal equations, their solutions and the computation of the residuals (corrections) (See section (8)).

The F or the chi-square test is used to test the a posteriori variance of unit weight or the reference variance ($\hat{\sigma}_0^2$) in order to assure that there is no significant difference

between the $\hat{\sigma}_0^2$ and the a priori variance of the unit weight σ_0^2 , which is assumed, in this paper, as being :

$$\sigma_0 = 4\text{mm / km}$$

If there is significant difference between $\hat{\sigma}_0^2$ and σ_0^2 then some of the estimates used to derive the observation equations must be wrong. The wrong observations are detected by appropriate statistical tests as they are given in detail (Baarda (41) and (42)). This blunder error detection of each observations can be performed as follows :

$$V_i \leq 1.96 \hat{\sigma}_0 \quad (17)$$

at 95% confidence level .

If the mathematical model of the network and the a priori estimation of the accuracy of observations has been

correct, the calculated $\hat{\sigma}_0^2$ value should agree with a priori accepted σ_0^2 value within the confidence interval at a specified probability level. Chi-square (χ^2) distribution is used for testing the agreement of the value $\hat{\sigma}_0^2$ with σ_0^2 by using the confidence interval. The test is performed as follows (Uotila (5); Mikhail (14)):

$$\frac{(n-u) \hat{\sigma}_0^2}{\chi_{p2}^2} \leq \sigma_0^2 \leq \frac{(n-u) \hat{\sigma}_0^2}{\chi_{p1}^2}$$

With χ_{p2}^2 and χ_{p1}^2 found from the statistical tables for the chi-square distribution at the required probability level (Mikhail (15, 16)). The values of χ_{p2}^2 and χ_{p1}^2 for 95% and 99% probability level is a function of degrees of freedom DF (difference n-u) in the network. If the number of degrees of freedom is very small, the

certainty attached to estimated $\hat{\sigma}_0^2$ is, of course, very small. This is reflected by a very large confidence interval for a priori value of σ_0^2 . The variance-covariance (dispersion) matrix is also calculated . It is very useful tool to test the strength of a survey network for planning a new network or for revising an existing one (Bomford (1) and Mikhail et al (2 , 3)).

The symmetrical matrix

$$\Sigma_{xx} = \hat{\sigma}_0^2 N^{-1} \quad (19)$$

The variance-covariance matrix of unknowns is given by the product of the inverse of normal equations (N^{-1}) by the unit variance ($\hat{\sigma}_0^2$). The matrix Σ_{xx} is a square and symmetrical. the diagonal terms of the matrix Σ_{xx} are the variances of the corresponding unknowns, and the non-diagonal terms are the covariances between the unknowns. The matrix Σ_{xx} is a function of the weights (or precision) of the field survey data and the shape of the network, as defined

by the A matrix.

The least squares adjusted differences in height for each level line are :

$$\hat{h}_i = h_{oi} + V_i \quad (20)$$

The next stage of the computations is to calculate the standard error of the measured differences in height as follows :

$$\sigma_{\Delta_{hi}} = \hat{\sigma}_0 \sqrt{1/W_i} \quad (21)$$

Where : $\sigma_{\Delta_{hi}}$ = the standard error of a weighted value, and W is the weight. $\hat{\sigma}_0$ = is the standard error of unit weight, $1/W_i$ = is the reciprocal of the weight, is referred to as a weight number or cofactor.

The final stage of the calculations is to evaluate the accuracy of the adjusted heights of the junction points. However, there are two methods for computing the standard error of the adjusted heights of the junction points (σ_{zi}) as follows :

a - From the inverse matrix of the normal equations (N)

$$\sigma_{zi} = \hat{\sigma}_0 \sqrt{q_{ii}} \quad (22)$$

Where σ_{zi} = the standard error of the adjusted heights of the junction points σ_o = the a posterior standard error of unit weight q_{ii} = the diagonal terms of the inverse matrix (N^{-1})

b - From the variance-covariance matrix (Σ_{xx})

$$\sigma_{zi} = \sqrt{q_{ii}} \quad (23)$$

Where : q_{ii} = the diagonal terms of the variance-covariance matrix (Σ_{xx})

It should be noted that the above adjustment is based on the assumption that the corrections dz are small. If the approximate coordinates, obtained from preliminary computations, differ considerably from the adjusted values, the result of the adjustment will not be correct.

If there is any doubt about the quality of the approximate coordinates, the adjustment should be repeated, with the coordinates calculated the first adjustment as the approximate ones.

The number of iterations necessary to reduce dz to give a satisfactory solution will depend on the accuracy of the provisional coordinates at the outset. The least accurate these provisional coordinates, the longer will be the computational procedure. However, more than two iterations might indicate of either poor field data or input errors.

6. The designed computer program (ADJLVL)

A computer program named ADJLVL based on the proceeding analysis (See Section (5)) has been designed by the author. ADJLVL is coded in FORTRAN IV language and it has 365 lines including the comment lines. The computer program consists of a series of small and well-defined modules (subroutines and functions) to allow easy checking, handling and modifications. A small main program was designed to tie the required subroutines. These subroutines can be executed as one computer run, provided no errors occur in the input data or during the execution of the program. Data preparation for the current computer program was discussed in Section (3). In this section, the capabilities of the different subroutines are discussed briefly with the mathematical equations on which they are based (See Section (5)). A generalized flow chart indicates the main stages of the program adjustment operations is shown in Figure (1). An example of the input data is given in Appendix - A.

The current computer program based on the least squares adjustment (observation method) (variation of coordinates method). The program calculates the adjusted heights of a number of junction points for two cases :

1. when the measured differences in height are uncorrelated and have the same weight (identity weight matrix).
2. when the measurements are uncorrelated but have weight that are inversely proportional to the respective levelled distances.

The program first reads the necessary data i.e. the total number of fixed bench marks in the network, number of junction points, and the number of leveling routes. The program also reads the fixed stations with their heights, and the junction

stations with their provisional heights. The input data for each level (route) lines are read in such as the direction

of each level line (start station to end station), differences in height, length of the lines, and other control parameters (flags) to indicate whether the stations are fixed bench marks or new junction points (See Figure (A.1) and TABLE (A.1)).

The program forms up the observation equations matrix ($AZ + L = V$) with weight matrix W (See Eqn.(4) and Eqn. (5)).

Each observation equation is multiplied by the square root of the corresponding weight (See Eqn. (6) and Eqn. (9)), so that the multiplication by square root of the weight reduces all equations to zero dimension. Then, The zero columns of the observation equations are removed.

Subroutine (NORMAL) was called to form the normal equation matrix ($NZ = K$) (Eqn. (12)).

Subroutine (CHLSKY) was designed to solve the normal equations by the Cholesky's method. (i.e. $Z = -N^{-1}k$ (Eqn. (14)); the subroutine (INVRS) is used to invert the matrix N , the first step is form the upper triangular U . The matrix ' U ' is inverted then post- multiplied by its transpose to get N as shown below :

$$N = U^T U \quad (24)$$

$$N^{-1} = U^{-1} (U^{-1})^T \quad (25)$$

Because of the symmetry of N matrix, the U matrix is only stored in order to save core storage space in the computer.

Subroutine (COMPUT) calculates the final heights of the junction stations.

Subroutine (RESFUL) was designed to determine the residuals of the observations and the standard error of unit weight (Eqn. (15)).

The adjusted of the differences in height with their standard errors for each level line was also determined using Eqn. (20) and Eqn. (21).

Since the weights were calculated from Eqn. (9) with S in kilometers, the obtained value of σ_o represents an estimated standard deviation of levelling per kilometer. The result shows that the actual accuracy of the observations is slightly lower than the aforementioned expected accuracy of 4 mm/km.

However, the chi-square (χ^2) test (See Eqn. (18)) at 95% probability level for $DF = 10$ gives 3.7 mm $\leq \sigma_o \leq 9.4$ mm ; therefore, the survey may be accepted.

Thereafter, the standard errors of the adjusted heights of the junction stations are evaluated.

Subroutine (INVERSE) was used to form the inverse matrix of the normal equations (N^{-1}). (Eqn. (24) and Eqn. (25)).

Finally, the variance-covariance (dispersion) matrix (Σ_{xx}) are determined by using the subroutine (VARCOV) (Eqn. (19)).

The number of iteration in this example is one. In some cases, more than one iteration is required. This will depend on the accuracy of the provisional coordinates.

The output of the ADJLVL consists of a list of the input data, the statistics of the solution of the problem, the corrections (residuals) to the observations, the final adjusted coordinates of the stations and their covariance, and standard error of unit weight of the network (See Table (A. 2)).

7. Practical Results

The field leveling observations was taken from reference (27) which are used as a source of data in order to test the current adjustment program. Other sources of data are also used from engineering projects (sewerage project for Dooz area).

In appendix A are computer output in which are shown the adjustment results as they are provided by the program ADJLVL (See Table A. 2).

One might notice the simplicity of the input arrangement and the valuable information provided by the output of the computer program. This unified simple way to arrange the Input and output data , is applied to most of the vertical control networks independently from the geometric configuration of the network. This program can be used to adjust the First , second ,third and fourth order accuracy of the leveling networks.

8. Program Testing

The general procedure for testing a computer program can be found in various literatures on computer software Kernighan et al (43), Kernighan et al (44) and Yohe (45). However, for the current program, all program errors (Fortran, execution, algorithm and input data) have been detected and corrected during the program design and development with the aid of the compilers 90. Moreover, diagnostic error messages have also been incorporated into the program for checking several situations when something is wrong with the input data and the program. They are self-explanatory.

Beside testing the validity of input data from mistakes, the following arithmetical checks have been made :

1. The formation of the observation equations can only be checked by comparison between hand computation (for example, using a top desk calculator) with a computer output.

2. The formation of normal equations, their solutions and the calculation of the residuals can be checked by $A^T W V = 0$ (Eqn. (16)).

3. The formation of the inverse of normal equations matrix must be symmetrical (i.e. $q_{ij} = q_{ji}$).

A sample of the program output was checked with manual computation (e.g. using top desk calculator) and the results are agreed within the specified accuracy.

9. Program restrictions and storage

Every computer program has data limits and programs vary in how easy it is to change the more restrictive ones. The limits in the current program may be increased by

changing the size of the arrays, if desired. The following are the maximum values of the parameters which are used :

Maximum number of the fixed bench marks = 50

Maximum number of the junction stations = 50

Maximum number of the leveling (route) lines = 50

Maximum storage capacity for 50 stations = 18 K (octal) core memory storage.

Maximum space in the computer memory is occupied by the $(A^T W A + W)$ matrix which for 50 points (50 Z – coordinate) requires $50 (50 + 1) / 2 = 1275$ locations.

The processing time $\cong$ 6 Seconds.

10. Conclusions

A simple algorithm provides least squares solution to a levelling network has been presented.

This algorithm is of universal use in engineering surveying projects as it is independent of the geometric configuration of the network. Redundant observations of heights can be effectively used in a rigorous adjustment to increase the precision. The developed algorithm is capable of covering areas in engineering surveying, low order vertical control networks for roads, sewage and irrigation engineering projects and other fields (e.g geological schemes).

Although non-rigorous or empirical methods applied in the adjustment of a leveling network give satisfactory results, it is better to use rigorous solutions which provide a state of the art process.

The rigorous method, as presented in this paper, has insignificant processing costs, it is easier to apply and it provides all desirable information about the adjustment :

1- Statistics of the stochastic model, 2- adjusted height coordinates of all stations in the levelling network and their variances, 3- adjusted differences in heights and their variances. Further studies might also be done to the use of digital level , where the data can be stored in some form of data collector in order to eliminate the errors or mistakes during the manual field method.

Recommendation will include the

automatic approximate coordinate generation. Moreover, the Chi-square test (χ^2) and the blunder detection and analysis techniques should be included in the algorithm . Also, the computer program can be translated from FORTRAN IV to FORTRAN 90 in order to reduce the size of the program, the processing time and storage capacity .

APPENDIX A

Results from the adjustment of the vertical control network shown in Figure (3) A.1 Data preparation All input data are punched via a computer terminal. Each of the following cards will represent a data card (a teletype line on a terminal screen). The following items of data being included :

Input data for program ADJLVL (See Figure (A.1))

CARD 1	NPTS = Number of the fixed bench marks NDIST = Number of the level lines in the network MFIK = Number of the new junction points MBEN = Number of the fixed bench marks in the network
CARD 2	IBEN = Reference number of the fixed bench marks ZBEN = Fixed height of the bench mark Card 2 is repeated for each of ' MBEN ' values
CARD 3	ITZ = Reference number of the new junction points Z = Computed height (Z- Coordinate) of the new junction points Card 3 is repeated for each of ' ITZ ' values.
CARD 4	NZ = Code number of the level line IFM = Reference number of the start station of a line IJ IT = Reference number of the finish station of a line IJ ITYPEI = Code number of the start station of a line IJ ITYPEJ = Code number of the end station f a line IJ DIST = Length of the level line IJ in Kilometres HC = Computed (Provisional) difference in height between the start and finish stations of the level line IJ (m) HO = Observed difference in height between two the start and end stations of the line IJ (m.)
Note 1	ITYPEI = 00 if the station is fixed bench marks ITYPEI = 10 if the station is new junction station ITYPEJ = 00 if the station is fixed bench marks ITYPEJ = 10 if the station is a new junction points Card 4 is repeated for each of ' NDIST ' values

TABLE (A.1) Input data file for the program " ADJLVL "

8	14	4	4				
1	15.263						
2	14.832						
3	13.715						
4	12.826						
5	14.358						
6	14.001						
7	14.229						
8	13.722						
1	3	7	10	0	2.0	0.514	0.520
2	7	8	00	0	1.5	-0.507	-0.506
3	4	8	10	00	1.8	0.896	0.890
4	4	6	10	00	2.5	1.175	1.187
5	6	5	00	00	1.7	0.357	0.365
6	3	5	10	00	2.4	0.643	0.638
7	1	7	10	00	2.2	-1.034	-1.041
8	7	6	00	00	2.0	-0.228	-0.235
9	2	6	10	00	0.8	-0.831	-0.828
10	1	5	10	00	1.0	-0.905	-0.906
11	5	8	00	00	1.8	-0.636	-0.628
12	2	8	10	00	2.1	-1.110	-1.120
13	5	7	00	00	0.8	-0.129	-0.130
14	8	6	00	00	1.0	0.279	0.278

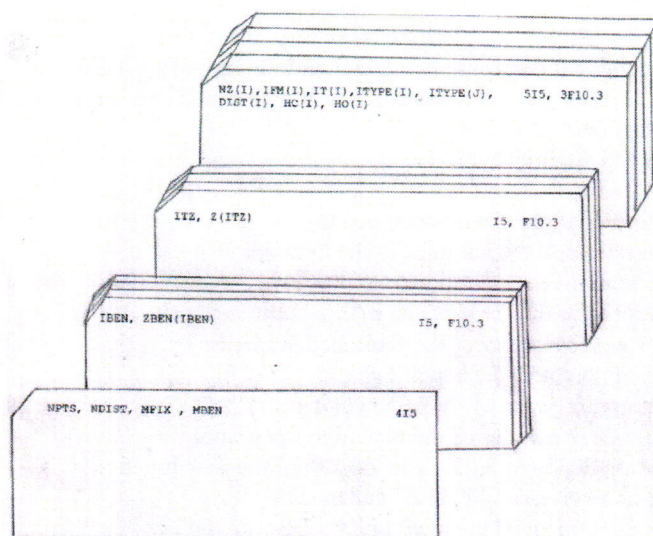


FIGURE (A.1) A simplified representation of the input data
format for the program " ADJLVL "

TABLE (A.2) Typical output results for the program " ADJLVL "

THE HEIGHTS OF THE FIXED BENCH MARKS

FIXED BENCH MARKS	HEIGHTS (M)
1	15.263
2	14.832
3	13.715
4	12.826

MATRIX OF OBSERVATION EQUATIONS WITH ZERO COLUMNS REMOVED

	1	2	3	4	5	6	
ROW NO. 1							
	.000	.000	.707	.000	-4.243	.500	
ROW NO. 2							
	.000	.000	-.816	.816	-.816	.667	
ROW NO. 3							
	.000	.000	.000	.745	4.472	.556	
ROW NO. 4							
	.000	.632	.000	.000	-7.589	.400	
ROW NO. 5							
	.767	-.767	.000	.000	-6.136	.588	
ROW NO. 6							
	.545	.000	.000	.000	3.227	.417	
ROW NO. 7							
	.000	.000	.674	.000	4.719	.455	
ROW NO. 8							
	.000	.707	-.707	.000	4.950	.500	
ROW NO. 9							
	.000	1.118	.000	.000	-3.354	1.250	

ROW NO. 10
 1.000 .000 .000 .000 1.000 1.000

ROW NO. 11
 -.745 .000 .000 .745 -5.963 .556

ROW NO. 12
 .000 .000 .000 .690 6.901 .476

ROW NO. 13
 -1.118 .000 1.118 .000 1.118 1.250

ROW NO. 14
 .000 1.000 .000 -1.000 1.000 1.000

MATRIX OF NORMAL EQUATIONS

ROW NO. 1
 3.810458 -.588235 -1.250000 -.555556 1.571895

ROW NO. 2
 -.588235 3.738235 -.500000 -1.000000 .655882

ROW NO. 3
 -1.250000 -.500000 3.371212 -.666667 -1.401515

ROW NO. 4
 -.555556 -1.000000 -.666667 3.253968 1.984127

SOLUTION OF NORMAL EQUATIONS

X(1)= -.653 MM.

X(2)= -.531 MM.

X(3)= -.083 MM.

X(4)= -.901 MM.

THE FINAL ADJUSTED HEIGHTS OF THE JUNCTION STATIONS

JUNCTION STN	HEIGHTS (M)
5	14.357
6	14.000
7	14.229
8	13.721

THE RESIDUALS

V(1)= -6.083 MM.
 V(2)= -1.818 MM.
 V(3)= 5.099 MM.
 V(4)= -12.531 MM.
 V(5)= -8.123 MM.
 V(6)= 4.347 MM.
 V(7)= 6.917 MM.
 V(8)= 6.553 MM.
 V(9)= -3.531 MM.
 V(10)= .347 MM.
 V(11)= -8.248 MM.
 V(12)= 9.099 MM.
 V(13)= 1.570 MM.
 V(14)= 1.371 MM.

CHECK = ATWV = .000000

LINE NO.	ADJUSTED DH S (M)	STANDARD ERROR OF THE DH S (MM.)
1	.516	7.560
2	-.507	6.547
3	.894	7.172
4	1.179	8.452
5	.359	6.970
6	.641	8.281
7	-1.036	7.928
8	-.230	7.560
9	-.832	4.781
10	-.906	5.345
11	-.634	7.172
12	-1.114	7.746
13	-.128	4.781
14	.279	5.345

STANDARD ERROR OF OBSERVATION OF UNIT WEIGHT = 5.3 MM. PER KM.

INVERSE OF NORMAL EQUATIONS MATRIX

ROW NO. 1	.357	.115	.176	.132
ROW NO. 2	.115	.342	.123	.150
ROW NO. 3	.176	.123	.410	.152
ROW NO. 4	.132	.150	.152	.407

THE ACCURACY OF THE ADJUSTED HEIGHTS OF THE JUN.STN

JUNNCTION STN STANDARD ERROR (MM.)

5	3.2
6	3.1
7	3.4
8	3.4

THE VARIANCE-COVARIANCE MATRIX

ROW NO. 1				
	10.204	3.288	5.019	3.781
ROW NO. 2				
	3.288	9.778	3.517	4.287
ROW NO. 3				
	5.019	3.517	11.716	4.338
ROW NO. 4				
	3.781	4.287	4.338	11.633

References

1. BOMFORD, G., "Geodesy", 3 rd edition, Oxford, 1977.
2. BOMFORD, G., " The geodetic adjustment of Australia , 1963-1966, Survey Review, Vol. XIX, No. 144, April, 1967.
3. ASHKENAZI, V. D., "Solution and error analysis of large geodetic networks, Survey Review, Vol. XIX, No. 146 and 147, 1967.
4. ASHKENAZI, V. D., " Adjustment of control networks for precise engineering surveys ", Chartered Surveyor, No. 102, 1970 .
5. UOTILA, U. A., "Statistical tests as guidelines in Analyses of adjustment of control nets ", Surveying and Mapping, ACSM, March, 1975.
6. BARBER, J. D., FULFORD, F. B. and DHALLA, Z. K., " Computer program MANOR (Multipurpose Analyses of Network Observations and Reductions)", The Canadian Surveyor, Vol. 33, No. 3, Sept. 1979.
7. CLARK, D. and JACKSON, J. E., "Plane and Geodetic Surveying for engineers ", Vol. 2, 6th edition, Constable & Company Ltd., London, 1973.
8. HAZAY, I. , "Adjusting Calculations in Surveying", AKADEMIAT, KIADO, Budapest, 1970
9. RAINSFORD, H. F., "Survey Adjustments and Least Squares ", Constable and Co., London, 1957.
10. ALLAN, A. L. , HOLLWEY, J. R. , and MAYNES, J. H. B. , " Practical Field Surveying and Computations ", American Edition, New York, American Elsevier, 1968.
11. RICHARDUS, P., "Project Surveying", Amsterdam, North -Holland, 1966.
12. MILNE, P. H. , " Basic Programs For Land Surveying ", London, New York, E & F.N. Spon, 1983
13. MOFFITT, F. H. , and BOUCHARD, H. , "Surveying ", Harper & Row, Publishers, New York, 1987.
14. MIKHAIL, E. M. , "Observations and least squares ", New York, Harper & Row., 1976.
15. MIKHAIL, E. M., AND GRACIE, G., "Analysis and adjustment of survey measurements", New York, Van Nostrand Reinhold, 1981.
16. MIKHAIL, E. M., and ACKERMANN, F., "Observations and least squares" , IEP-A Dunn-Donnelly Publisher, New York, 1983.
17. WOLF, P. R. , " Adjustment Computations (Practical Least Squares for Surveyors) ", 2nd edition, P.B.L. Publishing Co., Monona, Wisconsin, 1980.
18. SCHOFIELD, W. , " Traverse Adjustment by variation of co-ordinates ", The Civil Engineering Surveyor, June and Sept. 1979 .
19. KAVANAGH, B. F., "Surveying: Principles and Applications", 7th edition, Pearson, Prentice Hall, New Jersey, United States of America, 2006.
- 20 . NATHANSON, J. A., LANZAFAMA, M. T., and KISSAM , P., " Surveying Fundamentals and practices ", 5th edition, Pearson, Prentice Hall, New Jersey, 2006.

21. ATKINSON, N. and ALLAN, A.L., "Back to Basic (24) Least Squares, Statistics and All That", SURVEY REVIEW , Vol. 36, No. 282, October 2001 .
22. THOMPSON, E . H., "The theory of the method of least squares, Photogrammetric Record, Vol. IV, NO. 19, April 1962.
- 23 . GALE, L. A. , "Theory of adjustment by least squares , The Canadian Surveyor , VOL. XIX , March, 1965.
24. DEUMLICH, F., "Instrumentenkunde der Vermessungstechnik, English translated edition, Berlin, 1972.
25. KOWALCZYK, Z., "Geodesy, VOL. 2: Levelling, U. S. Dept. of Commerce, 1968
26. COOPER, M. A. R., " Modern Theodolites and Levels " , Crosby Lockwood & Son Ltd., London , 1971
27. HAUSBRANDT, S. "Rachunek Wyrownawczy i Obliczenia Geodezyjne (Adjustment Calculus And Surveying Computations, English translated edition) , Panstwowe Przedsiębiorstwo Wydawnictw Kartograficznych , Warsaw, 1971.
28. Surveys and Mapping Branch, Canada, " Specifications and Recommendations for Control Surveys and Survey Markers ", Ottawa, 1973.
29. DAY, A. C. , " Non-numerical techniques " , Computer Centre, University College London, Technical Report, No.1, January, 1971.
30. DAY, A. C. , " Fortran Techniques " , CUP, University College London, 1972.
31. DAVID, D. and Fisher, K. P. , " Land Parcel Identifiers for Information Systems, Analysis of the Conference on Land Identifiers- The Problem, Prospects and Payoffs, American Bar Foundation, Chicago, 1973.
32. BOGARETS, M . J. M. , " Land Data Systems " , XV FIG., International Congress of Surveyors, Commission 5, Paper No. 508.1, Sweden, June 1977.
33. BLACHLUT, T. R., CHRZANOWSKI, A. and SAASTMOIONEN, J. H. , " Urban Surveying and Mapping " , Springer-Verlag, New York Inc., 1979.
34. PALMER. A. H., "Computer Aided Land Surveys", Chartered Land Surveyor / Chartered Minerals Surveyors , Vol. 2, No. 3, Winter , 1980 .
35. JOHNSTON, W. F., "Benefits and Problems of In-house Programming", Civil Engineering Surveyor, Vol. 7, Issue &, September , 1980.
36. MILS, D. J., and PALMER, A. H., "Computing in Construction Surveying, Paper presented at the Institution of Civil Engineering Surveyors Symposium, Loughborough, April , 1982.
37. SHERWOOD, P. C., "Computers in Engineering Surveying " , Civil Engineering Surveyor, VOL. VII, Issue6, July / August 1982.
38. RICE , J. R., " Numerical Methods, Software, and Analysis " , McGraw – Hill Book Co., Singapore, 1983.
39. THOMPSON, E. H., " An introduction to the algebra of matrices with some applications", Adam Hilger, London , 1969.
40. BJERHAMMAR, A., "Theory of Errors and Generalized Matrix Inverses", Elsevier Scientific publishing Company, Amsterdam - London – New York, 1973 .
41. BAARDA , W., " Statistics : A Compass for the Land Surveyor", Paper presented at XIIth FIG International Congress of Surveyors, Commission 5, Paper No. 507 (C), London, 1968.
42. BAARDA, W., " A Testing Prcedure for Use in Geodetic Network", Netherlands Geodetic Commission, Publications in Geodesy, New Series, Vol. 2, No. 5, Delft, 1968.
43. KERNIGHAN, B. W., and PLAUGER, P. J., "Programming style : examples and counter examples, Computing Surveys, Vol. 6, No. 4, December 1974 .
44. KERNIGHAN, B. W., and PLAUGER, P. J., "The elements of programming style " , 2nd Edition, McGraw-Hill , 1978.
- 45 . YOHE, J. M., " An overview of programming practices " , Computing Surveys, Vol. 6, No. 4, December, 1974.

خوارزمية بسيطة لحل شبكات التسوية بطريقة المربعات الصغرى

سامي حميد علي

كلية الهندسة ، جامعة تكريت ، تكريت ، العراق

الملخص

يعتمد البحث السمات النظرية والعملية لحل شبكات التسوية بطريقة المربعات الصغرى بأسلوب خوارزمية بسيطة والتي تم تطويرها في هذا البحث .
بيانات شبكات التسوية تتضمن مسح حقل باستخدام طريقة قياسات فرق المنسوب (أي فرق الارتفاع بين رواقم التسوية الثابتة والنقاط الجديدة المستحدثة) والتي يمكن أن تصحح بواسطة طريقة المربعات الصغرى بدون زيادة في الكلفة مقارنة مع طرق التصحيح التقريبية.
الخوارزمية تعتمد على طريقة (التغير في الإحداثيات) (طريقة القياسات). المخرجات النهائية للبرنامج الحسابي تتضمن تحليل مصفوفة التباين والتباين المشترك .

بالرغم من بساطة الخوارزمية ، فإنها تزودنا بقدرة الاستعمال العام في قياس وحساب الارتفاعات المصححة لرواقم التسوية الجديدة المستحدثة في مجال المساحة الحقلية المستوية والتي تستخدم في مشاريع الطرق ، المجاري ، الري ، والجيولوجي ، وكذلك المشاريع الأخرى للهندسة المدنية والبيئة .